

**Ph195b Final EXAM**

**Due Wednesday 20 March at 5:00pm, in the box outside 24 Bridge Annex.**

3 hour time limit, one page of your own notes only.

1. (20 points) Find positive-energy stationary solutions of the Schrödinger Equation for the 1-D potential

$$V(x) = g\delta(x), \quad x < +L,$$

$$= +\infty, \quad x \geq +L.$$

Assume  $g$  and  $L$  are real positive constants.

2. Consider the following potential in 2-D:

$$V(x,y) = \sum_{q=1}^4 g \delta(x - \cos(q\pi/2)) \delta(y - \sin(q\pi/2)).$$

- a. (10 points) Identify a discrete symmetry of the 2-D Schrödinger Equation with this potential, and write down the corresponding differential operator in the position representation (you are free to choose convenient coordinates).
- b. (10 points) Show that this operator commutes with the total Hamiltonian. You may treat the action of this operator on the delta-functions informally, but please be explicit about your reasoning.
- c. (20 points) Write down a class of eigenstates of the 2-D Schrödinger Equation with this potential. (Hint: consider solutions whose energies satisfy  $E = p^2/2m$ .)

3. Consider an addition of angular momentum scenario with  $J_A = 1$  and  $J_B = 1$ . Note that a table of Clebsch-Gordan coefficients is included with this exam.

- a. (5 points) What are the possible values of  $J_{tot} \equiv J_A + J_B$ ?
- b. (10 points) Rewrite the state

$$|\Psi_{AB}\rangle = \frac{1}{\sqrt{2}} (|J_A = 1, m_A = +1\rangle + |J_A = 1, m_A = -1\rangle) \otimes |J_B = 1, m_B = +1\rangle$$

in the  $\{|J_{tot}, m_{tot}\rangle\}$  basis. Do not leave any undetermined coefficients.

c. (10 points) Suppose our quantum system consists of two spin-1 particles. Show that the joint state  $|J_{tot} = 0, m_{tot} = 0\rangle$  is an entangled state of the two spins.

d. (15 points) Re-express your answer to part (b) in a coordinate system that is rotated by angle  $\alpha$  about the  $x$ -axis. You may wish to make use of the general expression

$$\mathbf{J}_{\pm}|j; m\rangle = \sqrt{(j \mp m)(j \pm m + 1)} \hbar |j; m \pm 1\rangle.$$

Total possible points = 100.

Table 8.11.  
Numerical Values of the Clebsch-Gordan Coefficients.

$a$	$\alpha$	$b$	$\beta$	$C_{\alpha\alpha b\beta}^{0\alpha+\beta}$		$a$	$\alpha$	$b$	$\beta$	$C_{\alpha\alpha b\beta}^{1/2\alpha+\beta}$	
0	0	0	0	1	1.000000	5/2	5/2	2	-2	$1/\sqrt{3}$	0.577350
1/2	1/2	1/2	-1/2	$1/\sqrt{2}$	0.707107	5/2	3/2	2	-1	$-2/\sqrt{3 \cdot 5}$	-0.516398
1	1	1	-1	$1/\sqrt{3}$	0.577350	5/2	3/2	2	-2	$1/\sqrt{3 \cdot 5}$	0.258199
1	0	1	0	$-1/\sqrt{3}$	-0.577350	5/2	1/2	2	0	$1/\sqrt{5}$	0.447214
3/2	3/2	3/2	-3/2	1/2	0.500000	5/2	1/2	2	-1	$-\sqrt{2/3 \cdot 5}$	-0.365148
3/2	1/2	3/2	-1/2	-1/2	-0.500000	3	3	5/2	-5/2	$\sqrt{2/7}$	0.534522
2	2	2	-2	$1/\sqrt{5}$	0.447214	3	2	5/2	-3/2	$-\sqrt{5/3 \cdot 7}$	-0.487950
2	1	2	-1	$-1/\sqrt{5}$	-0.447214	3	2	5/2	-5/2	$1/\sqrt{3 \cdot 7}$	0.218218
2	0	2	0	$1/\sqrt{5}$	0.447214	3	1	5/2	-1/2	$2/\sqrt{3 \cdot 7}$	0.436436
5/2	5/2	5/2	-5/2	$1/\sqrt{2 \cdot 3}$	0.408248	3	1	5/2	-3/2	$-\sqrt{2}/\sqrt{3 \cdot 7}$	-0.308607
5/2	3/2	5/2	-3/2	$-1/\sqrt{2 \cdot 3}$	-0.408248	3	0	5/2	1/2	$-1/\sqrt{7}$	-0.377964
5/2	1/2	5/2	-1/2	$1/\sqrt{2 \cdot 3}$	0.408248						
3	3	3	-3	$1/\sqrt{7}$	0.377964	$a$	$\alpha$	$b$	$\beta$	$C_{\alpha\alpha b\beta}^{1\alpha+\beta}$	
3	2	3	-2	$-1/\sqrt{7}$	-0.377964	1/2	1/2	1/2	1/2	1	1.000000
3	1	3	-1	$1/\sqrt{7}$	0.377964	1/2	1/2	1/2	-1/2	$1/\sqrt{2}$	0.707107
3	0	3	0	$-1/\sqrt{7}$	-0.377964	1	1	0	0	1	1.000000
						1	0	0	0	1	1.000000
$a$	$\alpha$	$b$	$\beta$	$C_{\alpha\alpha b\beta}^{1/2\alpha+\beta}$		1	1	1	0	$1/\sqrt{2}$	0.707107
1/2	1/2	0	0	1	1.000000	1	1	1	-1	$1/\sqrt{2}$	0.707107
1	1	1/2	-1/2	$\sqrt{2/3}$	0.816497	1	0	1	0	0	0.000000
1	0	1/2	1/2	$-1/\sqrt{3}$	-0.577350	3/2	3/2	1/2	-1/2	$\sqrt{3/2}$	0.866025
3/2	3/2	1	-1	$1/\sqrt{2}$	0.707107	3/2	1/2	1/2	1/2	-1/2	-0.500000
3/2	1/2	1	0	$-1/\sqrt{3}$	-0.577350	3/2	1/2	1/2	-1/2	$1/\sqrt{2}$	0.707107
3/2	1/2	1	-1	$1/\sqrt{2 \cdot 3}$	0.408248	3/2	3/2	3/2	-1/2	$\sqrt{3/2 \cdot 5}$	0.547723
2	2	3/2	-3/2	$\sqrt{2/5}$	0.632456	3/2	3/2	3/2	-3/2	$3/(2\sqrt{5})$	0.670820
2	1	3/2	-1/2	$-\sqrt{3/2 \cdot 5}$	-0.547723	3/2	1/2	3/2	1/2	$-\sqrt{2/5}$	-0.632456
2	1	3/2	-3/2	$1/\sqrt{2 \cdot 5}$	0.316228	3/2	1/2	3/2	-1/2	$-1/(2\sqrt{5})$	-0.223607
2	0	3/2	1/2	$1/\sqrt{5}$	0.447214	2	2	1	-1	$\sqrt{3/5}$	0.774597
						2	1	1	0	$-\sqrt{3/2 \cdot 5}$	-0.547723