

Ph195c Midterm EXAM

Due Tuesday 7 May at 5:00pm, in the box outside 24 Bridge Annex.

3 hour time limit, open notes (your own notes, class notes from Ph195abc, and supplemental readings).

1. Consider scattering from the spherical “shell” potential,

$$V(r) = +g\delta(r - a),$$

where g and a are positive real constants.

- (20 points) Compute the s -wave cross section σ_s as a function of incident energy E .
- (5 points) Is there any resonant behavior in $\sigma_s(E)$? Discuss.
- (5 points) Specify a condition on E such that the contribution of higher partial waves should be negligible.

2. Consider a spin- $\frac{1}{2}$ particle of mass m moving in three kinetic dimensions, subject to the spin-dependent potential

$$\mathbf{V}_1 = \frac{1}{2}k|-\rangle\langle -| \otimes |\vec{\mathbf{r}}|^2,$$

where k is a positive real constant, $\vec{\mathbf{r}}$ is the three-dimensional position operator, and $\{|-\rangle, |+\rangle\}$ span the spin part of the Hilbert space. Let the initial state of the particle be prepared as

$$|\Psi_0\rangle = |-\rangle \otimes |0\rangle,$$

where $|0\rangle$ corresponds to the ground state of the (harmonic) motional potential.

a. (20 points) Suppose that a perturbation

$$\mathbf{W} = \hbar\Omega(|-\rangle\langle +| + |+\rangle\langle -|) \otimes \mathbf{1}^{cm},$$

(where $\mathbf{1}^{cm}$ denotes the identity operator on the motional Hilbert space) is switched on at time $t = 0$. Using Fermi's Golden Rule, compute the rate of transitions out of $|\Psi_0\rangle$.

b. (5 points) Describe qualitatively the evolution induced by \mathbf{W} , in the limits $\Omega \gg \sqrt{k/m}$ and $\Omega \ll \sqrt{k/m}$. *Hint:* Make sure you understand part (c).

c. (5 points) Consider a different spin-dependent potential

$$\mathbf{V}_2 = |+\rangle\langle +| \otimes \Sigma_+(\mathbf{x}) + |-\rangle\langle -| \otimes \Sigma_-(\mathbf{x}),$$

where $\Sigma_{\pm}(\mathbf{r})$ denote the motional potentials

$$\begin{aligned} \Sigma_+(\mathbf{x}) &= +\infty, & |x| < a, \\ &= 0, & |x| \geq a, \\ \Sigma_-(\mathbf{x}) &= 0, & |x| < a, \\ &= +\infty, & |x| \geq a, \end{aligned}$$

and a is a positive real constant. Let the initial state of the system be prepared as

$$|\Psi_0\rangle = |-\rangle \otimes |0'\rangle,$$

where $|0'\rangle$ corresponds to the ground state of $\Sigma_-(\mathbf{x})$. Explain why Fermi's Golden Rule predicts vanishing transition rate for the perturbation \mathbf{W} specified in part (a) above.

3. (20 points) In your own words, provide brief physical interpretations (~a few sentences each) of the principle quantum number n , the five significant angular momenta, and the fine and hyperfine Hamiltonians for the hydrogen atom.