

The following problems are from Ch. 12 of Merzbacher

1. (Exercise 12.7) Show that a spherical square well

$$\begin{aligned} V(r) &= -V_0, & r < a \\ &= 0 & r \geq a \end{aligned}$$

has no bound state unless

$$V_0 a^2 > \frac{\hbar^2 \pi^2}{8m}.$$

2. (Problem 8) Calculate the probability distribution for the momentum of the electron in the ground state of a (non-relativistic, spinless) hydrogen atom. Calculate both $\langle x^2 \rangle$ and $\langle p_x^2 \rangle$ (where x is a Cartesian coordinate and p_x is its associated momentum) for this state, and verify that the uncertainty relation is satisfied.

3. (Problem 16) Using first-order perturbation theory, estimate the correction to the ground-state energy of a (non-relativistic, spinless) hydrogen atom due to the finite size of the nucleus. Under the assumption that the nucleus is much smaller than the atomic radius, show that the energy change is approximately proportional to the nuclear mean square radius. Evaluate the correction for a uniformly charged spherical nucleus of radius R . Is the level shift due to the finite nuclear size observable? Consider both electronic and muonic (one proton for the nucleus with one muon orbiting instead of one electron) atoms.