Ph195c, study problems for 4/15/02

1. (Merzbacher, Exercise 19.3) Using the first-order formula for transitions from a state s to states $k \neq s,$ calculate the probability that the system remains in states s. Show that the result agrees with $P_{s \leftarrow s}(t)$ calculated from $\langle s | \tilde{T}(t, t_0) | s \rangle$ only if the second-order terms are retained in the perturbation expansion of this diagonal matrix element.

2. Consider a 1D linear harmonic oscillator with Hamiltonian

$$\mathbf{H}_0 = \frac{\mathbf{p}^2}{2m} + \frac{1}{2} m\omega_0 \mathbf{x}^2,$$

prepared initially in its ground state. In this problem we wish to consider depopulation of the ground state by time-dependent perturbations.

For a time-dependent perturbation of the form

$$V(t) = V_0 \mathbf{x} \cos \omega t,$$

use the results of first-order time-dependent perturbation theory to argue that there is a resonance when $\omega = \omega_0$. What are the limitations of such an argument?

Using a similar approach, deduce the resonance condition for a perturbation of the form

$$V(t) = \Omega \mathbf{x}^2 \cos \omega t,$$

which represents periodic modulation of the spring constant. Would such a perturbation have a similar effect on a classical harmonic oscillator, starting in its ground state?