Ph195b Study problems for 1/28/02

1NS. Use the crazy algebraic approach to work out the matrix elements $\langle \Psi_n | \mathbf{x}^2 | \Psi_k \rangle$ where $|\Psi_n\rangle$ and $|\Psi_k\rangle$ are eigenstates of the harmonic oscillator. Check your answer by deriving the same matrix elements using creation and annihilation operators.

2. Use perturbation theory and the result of Problem #1 to compute the second-order correction to the ground-state energy when an initial Hamiltonian

$$\mathbf{H}_0 = \frac{\mathbf{p}^2}{2m} + \frac{1}{2} m\omega^2 \mathbf{x}^2$$

is perturbed by

$$\mathbf{W} = \alpha \mathbf{x}^2,$$

with α a positive real parameter. Compare this to the exact ground-state energy of $\mathbf{H}_0 + \mathbf{W}$.

3. Prove that the creation operator $\mathbf{a}^\dagger$ has no normalizable eigenstates, and explain the reason.

4. Using dimensional analysis, derive the characteristic momentum scale p_0 for a harmonic oscillator. What role does p_0 play in the eigensystem of the LHO?

5. Suppose we prepare a harmonic oscillator in the initial state

$$|\Psi(0)\rangle = \frac{1}{\sqrt{2}}(|\alpha\rangle + |-\alpha\rangle),$$

where $|\alpha\rangle$ denotes a coherent state with amplitude $\alpha = 10$ and $|-\alpha\rangle$ denotes a coherent state with amplitude $-\alpha = -10$. Plot the probability density

$$|\psi(x,t)|^2 = |\langle x | \Psi(t)\rangle|^2$$

at times $t = 0$, $t = \pi/4\omega$, $t = \pi/2\omega$, $t = 3\pi/2\omega$, and $t = \pi/\omega$.