

Ph195 study problems for 10/22/01

1. Back on the subject of non-orthogonal projectors, do you think these can exist in real (as opposed to complex) vector spaces?

2. Suppose we have two spins, whose states live in two-dimensional Hilbert spaces H_A and H_B . Let the initial joint state be an arbitrary pure state

$$|\Psi_{AB}\rangle = c_{00}|0_a 0_b\rangle + c_{01}|0_a 1_b\rangle + c_{10}|1_a 0_b\rangle + c_{11}|1_a 1_b\rangle,$$

which may or may not be entangled.

(a). First assume that both spins are in the same laboratory, so that we can imagine “directly” performing measurements of the *factorizable* joint observable

$$\mathbf{O}_{AB} = \mathbf{A} \otimes \mathbf{B},$$

where \mathbf{A} is an observable on H_A and \mathbf{B} is an observable on H_B . Assume that \mathbf{A} and \mathbf{B} have spectral decompositions

$$\mathbf{A} = \lambda_a^0 |0_a\rangle\langle 0_a| + \lambda_a^1 |1_a\rangle\langle 1_a|,$$

$$\mathbf{B} = \lambda_b^0 |0_b\rangle\langle 0_b| + \lambda_b^1 |1_b\rangle\langle 1_b|.$$

Use the corresponding spectral decomposition of \mathbf{O}_{AB} to derive expressions for the values and probabilities of the four possible outcomes in a measurement of \mathbf{O}_{AB} . Find expressions for the mean value $\langle \mathbf{O}_{AB} \rangle$ and standard deviation $\Delta \mathbf{O}_{AB}$.

(b). Now assume that the spins have been brought to distant locations (from one another), such that Alice has the spin from H_A in Los Angeles and Bob has the spin from H_B in Boston. Since the two spins are no longer co-located, Alice can only perform a measurement of \mathbf{A} on her spin (whose state is described by a reduced density operator $\tilde{\rho}_A$) while Bob performs an “independent” measurement of \mathbf{B} on his spin (whose state is described by a reduced density operator $\tilde{\rho}_B$). Alice and Bob each have two possible outcomes for their measurements, so there are four possible combinations:

$$(0_a, 0_b) \quad (0_a, 1_b) \quad (1_a, 0_b) \quad (1_a, 1_b).$$

How do the probabilities of these (computed in this fashion) compare with the expressions you derived in part (a)? If we associate the value $\lambda_a^0 \lambda_b^0$ with the combination of outcomes $(0_a, 0_b)$, etc., what are the mean value and standard deviation?

(c). Does it matter whether $|\Psi_{AB}\rangle$ is entangled or factorizable?

3. What’s the difference between entanglement and statistical correlation? For example, with our two coin example we could consider a quantum entangled state

$$|\Psi_{AB}\rangle = \frac{1}{\sqrt{2}}[|H_a H_b\rangle + |T_a T_b\rangle],$$

where H and T denote heads-up and tails-up “states” of a coin. As we discussed in class, we can see directly from the state that we will always measure both coins to have the same state, even though we cannot predict ahead of time which state that will be. Alternatively, we can think of a mixed-state preparation of two coins in which half the time we put them both heads up and half the time we put them both tails up. This can be represented by a density

operator,

$$\rho_{AB} = \frac{1}{2}[|H_a H_b\rangle\langle H_a H_b| + |T_a T_b\rangle\langle T_a T_b|].$$

This preparation has the same correlation property, but is not entangled. Can you think of a measurement procedure that distinguishes the two cases?