Ph195b – Study problems for 1/9/02

1. Starting from the Schrödinger Equation

$$i\hbar\frac{\partial}{\partial t}\psi(x,t) = \frac{-\hbar^2}{2m}\frac{\partial^2}{\partial x^2}\psi(x,t) + V(x)\psi(x,t),$$

where $V(x)$ is an arbitrary real-valued function of x representing potential energy (with $V^*(x) = V(x)$), derive the one-dimensional *continuity equation*

$$\frac{\partial\rho}{\partial t} + \frac{\partial j}{\partial x} = 0,$$

where

$$\rho \equiv |\psi(x,t)|^2,$$

$$j \equiv \frac{\hbar}{2mi}\left[\psi^*\frac{\partial\psi}{\partial x} - \left(\frac{\partial\psi^*}{\partial x}\right)\psi\right].$$

What does this equation imply in terms of conservation of probability?

2. Compute the momentum-space representation $\tilde{\psi}(p)$ of the Gaussian wave function

$$\psi(x) = \frac{1}{(\pi\sigma_x)^{1/4}}\exp\left(-\frac{x^2}{2\sigma_x}\right).$$

3. Compute the commutator $[\mathbf{x}^2, \mathbf{p}^2]$. Express your answer in terms of the operators **1**, **x**, and **p**.