Ph195b – Study problems for 2/27/02

Due Tuesday 2/20 at noon, in class or in the mailbox **outside the Instructor's office**.

1. Merzbacher, Chapter 11, Problem #3 (page 255): Explicitly work out the **J** matrices for $j = 1/2,\ 1,$ and $3/2.$

2. [Modified from Cohen-Tannoudji, Diu, and Laloe, Complement F_{VI}, Exercise #2a] Consider a physical system whose three-dimensional state space is spanned by simultaneous eigenvectors of $\mathbf{J}^2$ and $\mathbf{J}_z$,

$$|j;m_z\rangle = \{\,|1;-1\rangle, |1;0\rangle, |1;1\rangle\,\},$$
$$\mathbf{J}^2|j;m_z\rangle = j(j+1)\hbar^2|j;m_z\rangle,$$
$$\mathbf{J}_z|j;m_z\rangle = m_z\hbar|j;m_z\rangle.$$

Express in terms of the kets $|j;m_z\rangle$ the eigenstates common to $\mathbf{J}^2$ and $\mathbf{J}_x$, to be denoted by $|j;m_x\rangle$.

3. [Modified from Cohen-Tannoudji, Diu, and Laloe, Complement F_{VI}, Exercise #10b] Let $\vec{\mathbf{J}}$ be the (vector) angular momentum operator of an arbitrary physical system whose state vector is $|\Psi\rangle$. Let $\langle\vec{\mathbf{J}}\rangle$ be the mean value of the angular momentum of the system. The coordinate axes are chosen in such a way that $\langle\mathbf{J}_x\rangle = \langle\mathbf{J}_y\rangle = 0.$ Show that

$$(\Delta\mathbf{J}_x)^2 + (\Delta\mathbf{J}_y)^2 \geq \hbar|\langle\mathbf{J}_z\rangle|.$$

4. [Modified from Cohen-Tannoudji, Diu, and Laloe, Complement F_{VI}, Exercise #10d] Continuing with the definitions from Problem #4 above, assume that the system under consideration is a spinless particle for which $\vec{\mathbf{J}} = \vec{\mathbf{L}} = \vec{\mathbf{r}} \times \vec{\mathbf{p}}$. Show that it is not possible to have both $\Delta\mathbf{L}_x\Delta\mathbf{L}_y = \frac{\hbar}{2}|\langle\mathbf{L}_z\rangle|$ and $(\Delta\mathbf{L}_x)^2 + (\Delta\mathbf{L}_y)^2 = \hbar|\langle\mathbf{L}_z\rangle|$ unless the wave function of the system is of the form

$$\psi(r,\theta,\varphi) = F(r, \sin\theta\exp(\pm i\varphi)).$$

You may want to look at Merzbacher Figure 11.2 for some ideas, and the following expressions for $\mathbf{L}_+$ and $\mathbf{L}_-$ in spherical coordinates may be useful:

$$L_+ = \hbar\exp(i\varphi)\left(\frac{\partial}{\partial\theta} + i\cot\theta\frac{\partial}{\partial\varphi}\right),$$
$$L_- = \hbar\exp(-i\varphi)\left(-\frac{\partial}{\partial\theta} + i\cot\theta\frac{\partial}{\partial\varphi}\right).$$